# Summary Game Theory 1

# MSc Economics

# 2017 - 2018



## Lecture 1 (slide 1-22)

Example of signalling game: gazelle vs tiger.

* Gazelles jump up and down when a tiger approaches them. They do so in order to signal that they are so fit and strong that they do not need to run away.

Games of normal form:

* Are static: each player only moves once.
* Involve complete information: each player knows the payoff function of all players.

Prisoner’s Dilemma:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Player 2 | |
| Player 1 |  | Don’t confess | Confess |
| Don’t confess | -2, -2 | -10, -1 |
| Confess | -1, -10 | -5, -5 |

Cournot Oligopoly:

* firms who choose non-negative quantities.
* Inverse demand:
* Cost functions:
* Elements of the game:
  + Player set: {1,…,I}.
  + Strategy sets:
  + Payoff functions:

General components of the normal form game:

* Participating players.
* Players’ possible actions.
* Payoffs for each player for each combination of actions.

Notation of a normal form game:

Cartesian product is a set of strategy vectors.

* Example of strategy vectors:
* Example of a Cartesian product:

## Lecture 2 (23-37)

Strictly dominated:

* + For all

Weakly dominated:

* + For all
* And:
  + For at least one

Rational players will not use strictly dominated strategies.

Procedure of iteratively deleting dominated strategies:

1a Mark all dominated strategies of player 1.

1b Mark all dominated strategies of player 2.

2 Apply step 1a and 1b to the resulting game.

3 Repeat until you find a game without dominated strategies.

Dominance solvable: when all players are indifferent between all strategies that have survived the procedure.

The set of strategies that survive the procedure of iterative deletion of strictly dominated strategies does neither depend on the order in which strategies are deleted nor on the number of strategies that are

deleted in each step.

This is not true with regard to the elimination of weakly dominated strategies. That is, in this case the result can depend for instance on whether one deletes all or only some dominated strategies per step.

Nash Equilibrium:

* + For all
  + For all
* All players play a best-response against each other, that is: no one has an incentive to deviate.

## Lecture 3 (38-51)

Cournot competition: compete in quantities. Example:

* Best response:
* NE:
* Nash equilibrium:

Bertrand: compete in prices. Example:

* + Charging the monopoly price leads to maximum profit, as it charges below player *j* and maximizes profit through the monopoly price.
  + There is no best-response: it is always better to undercut player *j*.
  + The interval is on the upward sloping part of the profit function.
  + Charging a higher price will lead to 0 profit.
  + Charging the same price will lead to 0 profit.
  + Charging a lower price will lead to losses.
  + Charging a higher price will lead to 0 profit.
  + Charging the same price will lead to losses.
  + Charging a lower price will lead to losses.

Nash equilibrium in Bertrand:

* Paradox: p=c even if there are only 2 firms.

## Lecture 4 (slide 52-84)

Bertrand competition assumptions:

* Homogenous products.
* Symmetric costs.
* Unlimited capacity: everyone is able to satisfy the entire demand by themselves.

Product differentiation:

* Products are not assumed to be homogeneous anymore.
* Result: firms are able/allowed to charge a higher price in order to capture a part of the market.

Example of Bertrand price competition with differentiated products:



Mixed strategies:

|  |  |  |  |
| --- | --- | --- | --- |
| Player 1 |  | Player 2 | |
|  | F | C |
| N | 0,2 | 0,2 |
| E | -1,-1 | 1,1 |

Player 1’s mixed strategies:

Player 2’s mixed strategies:

Best response of player 1:

* Probability of playing N = 0 if player 2 plays F with probability < 0.5.
* Probability of playing N between 0 and 1 if player 2 plays F with probability = 0.5.
* Probability of playing N = 1 if player 2 plays F with probability > 0.5.

Best response of player 2:

* Probability of playing F = 0 if player 1 plays N with probability < 1.
* Probability of playing F between 0 and 1 if player 1 plays N with probability = 1.

Nash equilibrium:

* Pure strategies: (1,0), (1,0)
* Continuum of mixed strategies:

Definition of a mixed equilibrium:

1. Expected utility of strategy 1 that is being played with a positive probability is equal to the expected utility of strategy 2 that is being played with a positive probability.
2. Expected utility of strategies 1 and 2 have to be at least as large as the expected utility of strategy 3 that is being played with probability = 0.

## Lecture 5 (slide 85-102)

|  |  |  |  |
| --- | --- | --- | --- |
| Player 1 |  | Player 2 | |
|  | H | T |
| H | -1,1 | 1,-1 |
| T | 1,-1 | -1,1 |

* Pure strategy NE: none.
* Calculate the mixed strategy NE:
* Solution of the mixed strategy NE:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Player 1 |  | Player 2 | | |
|  | L | C | R |
| T | 2,0 | 1,1 | 2,2 |
| M | 1,3 | 0,2 | 1,0 |
| B | 1,4 | 2,1 | 3,3 |

Step 1: delete strategy M (strictly dominated by T).

Step 2:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Player 1 |  | Player 2 | | |
|  | L | C | R |
| T | 2,0 | 1,1 | 2,2 |
| B | 1,4 | 2,1 | 3,3 |

Step 3: delete strategy R (strictly dominated by C).

Step 4:

|  |  |  |  |
| --- | --- | --- | --- |
| Player 1 |  | Player 2 | |
|  | L | R |
| T | 2,0 | 2,2 |
| B | 1,4 | 3,3 |

Step 5: no NE in pure strategies.

Step 6: calculate mixed strategies NE.

Step 7: write down the Nash Equilibria.

|  |  |  |  |
| --- | --- | --- | --- |
| Player 1 |  | Player 2 | |
|  | L | R |
| T | 2,2 | 2,2 |
| B | 3,-1 | 0,0 |

Step 1: pure strategy NE: (T,R).

Step 2: find mixed strategy NE.

Player 2 is indifferent between his two pure strategies as long as player 1 plays T with probability 1.

Player 1 will play T as long as the expected payoff from playing T is at least as large as the expected payoff from playing B.

when

Step 3: write down all NE.

Battle of the Sexes:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Player 1 |  | Player 2 | | |
|  | B | S | X |
| B | 4,2 | 0,0 | 0,1 |
| S | 0,0 | 2,4 | 1,3 |

Step 1: pure strategy NE: (B,B) and (S,S).

Step 2: denote the number of strategies played with a positive probability for each player

|  |  |  |  |
| --- | --- | --- | --- |
|  | Player 1 | Player 2 | Number of cases |
| A | 1 | 1 | 6 |
| B | 1 | 2 | 6 |
| C | 1 | 3 | 2 |
| D | 2 | 1 | 3 |
| E | 2 | 2 | 3 |
| F | 2 | 3 | 1 |

A: pure strategy NE: (B,B) and (S,S)

B: no NEs because player 2 would never be indifferent between any of their strategies in a mixture.

* If player 1 plays B, then player 2 will always play B.
* If player 1 plays S, then player 2 will always play S.

C: similar to B.

D: no NEs because player 1 would never be indifferent between any of their strategies in a mixture.

* If player 2 plays B, then player 1 will always play B.
* If player 2 plays S, then player 1 will always play S.
* If player 2 plays X, then player 1 will always play S.

E.1: mixed strategy between B and S.

E.2 mixed strategy between B and X.

* satisfies the inequality.
* Solution of the mixed strategy NE:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Player 1 |  | Player 2 | | |
|  | B | S | X |
| B | 4,2 | 0,0 | 0,1 |
| S | 0,0 | 2,4 | 1,3 |

E.3: mixed strategy between S and X.

* Does not hold, as player 1 will always play S no matter whether player 2 plays S or X.

F: mixed strategies between B, S and X.

Step 3: write down all equilibria.

## Lecture 6 (slide 103-?)